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Isotopic-Spin Analysis of Possible *CP*-Violating Effects in $K \to 3\pi$.

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Isotopic-Spin Analysis of Possible CP-Violating Effects in $K \to 3\pi$.

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Summary. — CP-violating effects in the decay $K \to 3\pi$ are analysed in terms of isotopic-spin parameters. In particular, the interference term between K_S and K_L amplitudes, and the difference in partial rates in $K^\pm$ decay are considered. In general, the real part of the interference term is expected to be much smaller than the imaginary part for transitions with $|\Delta I| < \frac{5}{2}$. The success of the $|\Delta I| = \frac{1}{2}$ rule in K_L- and K^+-decay sets an upper limit on the imaginary part of the interference term for CP-violating amplitudes with $|\Delta I| = \frac{1}{2}$. This limit is probably too high to be useful, but a significant effect in the real part would probably indicate that amplitudes with $|\Delta I| > \frac{1}{2}$ are important. This conclusion could be checked by comparing the real part of the interference term with the differences in partial rates in $K^\pm$ decay; the two effects are directly related through coefficients depending on $|\Delta I|$ and the isotopic spin of the final state. In K^0 decay, $I = 1$ final states ($|\Delta I| < \frac{5}{2}$) may be distinguished from $I = 3$ final states ($|\Delta I| \geqslant \frac{5}{2}$) by comparing the imaginary parts of the interference terms in the $\pi^+\pi^-\pi^0$ and the $3\pi^0$ modes.

1. – Introduction.

Recent experimental results [1] on the decay rate for $K_L \to 2\pi^0$ indicate that the CP-violating amplitude for $|\Delta I| > \frac{1}{2}$ may be comparable with that for $|\Delta I| = \frac{1}{2}$. It is of interest to attempt isotopic-spin analyses in other

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(1) J. M. GAILLARD, F. KREINEN, W. GALBRAITH, A. HUSSRI, M. R. JANE, N. H. LIPMAN, G. MANNING, T. RATCLIFFE, P. DAY, A. G. PARHAM, B. T. PAYNE, A. C. SHERWOOD, H. FAISSNER and H. REITHLER: *Phys. Rev. Lett.*, **18**, 20 (1967); J. W. CRONIN, P. F. KUNG, W. S. RISK and P. C. WHEELER: *Phys. Rev. Lett.*, **18**, 25 (1967).

CP-violating effects. We shall discuss effects which could be relevant to high-precision experiments on $K \to 3\pi$. In Sect **2**, we discuss the interference term in the K^0, $\overline{K^0}$ complex, and in Sect. **3**, we relate this term to differences in partial rates for K^+ and K^- decays. In Sect. **4**, we derive the relations presented in **2** and **3**. We assume *CPT* invariance throughout.

2. – $K^0 \to 3\pi$.

Consider the decay rate for $K^0 \to 3\pi$ in the charge mode $\boldsymbol{k}$ and the energy configuration $\{\omega\}$. For a beam which is pure K^0 at $t = 0$ and normalized to unit flux, we have the decay rate as a function of time:

$$(1)\quad R^{(k)}(t,\omega) = \tfrac{1}{2}\{|a_S^{(k)}(\omega)|^2 \exp[-\Gamma_S t] + |a_L^{(k)}(\omega)|^2 \exp[-\Gamma_L t] + \exp[-\tfrac{1}{2}(\Gamma_S+\Gamma_L)t]\cdot$$
$$\cdot [2\,\mathrm{Re}\,(a_S^{(k)}(\omega) a_L^{(k)*}(\omega))\cos(\Delta m t) - 2\,\mathrm{Im}\,(a_S^{(k)}(\omega) a_L^{(k)*}(\omega))\sin(\Delta m t)]\}\,,$$

where $\Delta m = m_S - m_L$, and $a_i^{(k)}(\omega)$ is the appropriate amplitude for $K_i \to 3\pi$.

It is well known that the decay $K_1 \to 3\pi^0$ is forbidden by *CP*, but the decay $K_1 \to \pi^+\pi^-\pi^0$ is allowed in the final states $I = 0, 2$. These states are suppressed by the $|\Delta I| = \frac{1}{2}$ rule and/or potential barriers ([2]), but the corresponding amplitudes can be expected to be comparable to the *CP*-violating amplitudes ($I = 1, 3$). However, because of symmetry properties, these amplitudes will not contribute to the *interference* term in eq. (1), *integrated over all energies* ([3,4]). Therefore we shall consider only this term.

Neglecting second-order *CP*-violating effects, we may write

$$(2)\qquad a_S^{(k)}(\omega) = a_1^{(\cdot)}(\omega) + \varepsilon/2\, a_2^{(k)}(\omega)\,,$$

$$(3)\qquad a_L^{(k)}(\omega) = a_2^{(k)}(\omega)\,,$$

where ε is the *CP*-violation parameter in the K mass matrix as defined by WOLFENSTEIN ([5]): $\varepsilon \approx (|\varepsilon|/\sqrt{2})(1+i)$. K_1 and K_2 are orthogonal combinations

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([2]) See for example: T. D. LEE and C. S. WU: *Weak Interactions* (Second Section), *Ann. Rev. Nucl. Sci.*, **16**, Chapt. 9 (1966).

([3]) The $K_1 \to I = 0, 2$ amplitudes may of course interfere with the $K_2 \to I = 0, 2$ amplitudes, but for such terms to be important would require a very strong enhancement of these modes in the *CP*-volating transitions, which is physically unreasonable. Furthermore, this would imply a large $\pi^+\pi^-$ asymmetry in the K_L decay distribution.

([4]) The interference term as a measure of *CP* violation has also been discussed in a recent preprint by L. WOLFENSTEIN.

([5]) L. WOLFENSTEIN: *Nuovo Cimento*, **42**, 17 (1966). From experiments ([1]) there are two possible approximate values for $|\varepsilon|$: $|\varepsilon| \approx (5.7 \div 5.9)\cdot 10^{-3}$, $|\varepsilon| \approx (0.1 \div 0.6)\cdot 10^{-3}$.

of K^0 and $\overline{K^0}$ defined so that $K_2 \leftrightarrow 2\pi(I=0)$. The interference term is then given by

$$\int d\omega 2 \,\mathrm{Re}\,(a_S^{(k)} a_L^{(k)*}) = \frac{|\varepsilon|}{\sqrt{2}} \Gamma_2^{(k)} + I_+^{(k)}, \tag{4}$$

$$\int d\omega 2i \,\mathrm{Im}\,(a_S^{(k)} a_L^{(k)*}) = i \frac{|\varepsilon|}{\sqrt{2}} \Gamma_2^{(k)} + I_-^{(k)}, \tag{5}$$

$$I_\pm^{(k)} = \int d\omega (a_1^{(k)} a_2^{(k)*} \pm a_2^{(k)} a_1^{(k)*}), \tag{6}$$

$$I_+^{000} + I_+^{+-0} = 0. \tag{7}$$

Nonzero values of I_+ and I_- would provide direct evidence for an amplitude for $K_1 \to 3\pi$ in a final state with $CP = -1$.

It should be pointed out that it is somewhat ambiguous, once CP violation has been established in the K-decay system, to say in *which* decay channel(s) the violation occurs, because only *relative* phases between different K-decay amplitudes have physical significance. Thus the experiments of ref. [1] have established only the existence of a *phase difference* between the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ amplitudes in $K^0 \to 2\pi$. Similarly a nonzero values of I_- in eq. (5) would indicate the existence of a phase difference between $K^0 \to 2\pi(I=0)$ and $K^0 \to 3\pi$ amplitudes. A distinction as to which amplitudes actually violate CP is, in fact, purely conventional, unless there is some physically meaningful procedure for defining the phase of the CP operator.

For example, if all K-decays of a certain type, say $\Delta I = \frac{1}{2}$ transitions and all leptonic transitions, had a common phase, it would be reasonable to say that CP is violated only in nonleptonic transitions with $|\Delta I| > \frac{1}{2}$ [6]. Similarly, there could exist a choice of phase such that the violation occurs only in *parity-violating* decays [7,8]. Note that the alternative possibility, that the CP violation occurs only in *parity-conserving* decays [9,8], is ruled out by the experiments of ref. [1].

For convenience we adopt the phase convention introduced by Wu and Yang [10] and Wolfenstein [5], according to which the amplitude for $K^0 \to 2\pi$

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[6] T. N. Truong: *Phys. Rev. Lett.*, **13**, 358 a (1964); T. Bowen: *Phys. Rev. Lett.*, **16**, 112 (1966).

[7] F. Zachariasen and G. Zweig: *Phys. Rev. Lett.*, **14**, 794 (1965).

[8] N. Byers, S. W. MacDowell and C. N. Yang: *Proc. Seminar on High Energy Physics*, Int. Centre for Theor. Phys. (1965).

[9] W. Alles: *Phys. Lett.*, **14**, 348 (1965).

[10] T. T. Wu and C. N. Yang: *Phys. Rev. Lett.*, **13**, 380 (1964).

$(I=0)$ is taken to be real:

$$a[\mathrm{K}^0 \to 2\pi(I=0)] = a[\overline{\mathrm{K}^0} \to 2\pi(I=0)], \tag{8}$$

$$K_1 = (K_0 + \overline{K_0})/\sqrt{2}, \qquad K_2 = (K_0 - \overline{K_0})/\sqrt{2} \to 2\pi(I=0), \qquad CP\,\mathrm{K}^0 = \overline{\mathrm{K}^0}.$$

If for a suitable choice of phase the parity-conserving transitions $\mathrm{K}^0 \to 3\pi$ could be made to conserve CP, the amplitude for $\mathrm{K}^0 \to 3\pi$ as defined by the phase convention (8), would be characterized by a *constant phase*, and would obey the $|\Delta I| = \frac{1}{2}$ rule (which we assume good up to a few per cent for CP-conserving processes). In the event that the CP violation should occur only in parity-violating processes with $|\Delta I| > \frac{1}{2}$, this phase would vanish, and $I_{\pm} = 0$. The case of constant phase has simple experimental consequences and we shall discuss it first.

2·1. $|\Delta I| = \frac{1}{2}$, *constant phase.* – In the following, we neglect electromagnetic corrections, including mass differences in the invariant amplitudes. We define $\Gamma_i^{\prime(k)}$ and $I_{\pm}^{\prime(k)}$ as the widths and interference terms (eq. (6)) divided by the appropriate phase-space integrals.

It has been pointed out ([11,12]) that the $|\Delta I| = \frac{1}{2}$ rule and the assumption of a constant CP-violating phase φ, give the predictions

$$\frac{\Gamma_2^{\prime +-0}}{2\Gamma_+^{\prime 00+}} = \frac{\Gamma_2^{\prime 000}}{\Gamma_+^{\prime ++-} - \Gamma_+^{\prime 00+}} = \cos^2\varphi \leqslant 1\,. \tag{9}$$

For the interference terms we have the additional predictions ([12,13])

$$I_-^{\prime(k)} = 2i\,\mathrm{tg}\,\varphi\,\Gamma_2^{\prime(k)}, \tag{10}$$

$$I_+^{\prime(k)} = 0\,. \tag{11}$$

Equations (10) and (11) are in fact independent of the $|\Delta I| = \frac{1}{2}$ rule if all $\mathrm{K} \to 3\pi$ amplitudes have a common phase.

2·2. $|\Delta I| < \frac{5}{2}$, *arbitrary phase.* – If $|\Delta I| < \frac{5}{2}$ only the final state $I = 1$ contributes to the interference term. Barring an accidental cancellation between the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ contributions to I_-, one expects

$$|I_+^{(k)}| \ll |I_-^{(k)}|\,. \tag{12}$$

([11]) N. Cabibbo: *Proc. of Erice School* (1964), p. 284.
([12]) M. K. Gaillard: *Nuovo Cimento*, **35**, 1225 (1965).
([13]) S. L. Glashow and S. Weinberg: *Phys. Rev. Lett.*, **14**, 835 (1965).

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I_+ arises only from amplitudes which are nonsymmetric in pion momenta; for a given transition (ΔI fixed) the ratio $|I_+/I_-|$ should be of the order of the error made in assuming a linear decay distribution for the decay $K_L \to \pi^+\pi^-\pi^0$, *i.e.* a few per cent [14].

If the CP-violating transitions (as well as the CP-conserving ones) obey $|\Delta I| = \frac{1}{2}$, we have in addition to (12), the predictions

$$\Gamma_2'^{(+-0)} + \Gamma_2'^{(000)} \leqslant \Gamma_+'^{(00+)} + \Gamma_+'^{(++-)}, \tag{13}$$

$$\frac{|I_-'^{(000)} + I_-'^{(0+-)}|}{2(\Gamma_2'^{(+-0)} + \Gamma_2'^{(000)})} \leqslant \left[\frac{\Gamma_+'^{(00+)} + \Gamma_+'^{(++-)} - (\Gamma_2'^{(+-0)} + \Gamma_2'^{(000)})}{\Gamma_2'^{(+-0)} + \Gamma_2'^{(000)}}\right]^{\frac{1}{2}}. \tag{14}$$

To the extent that the squares of nonsymmetric amplitudes may be neglected (linear Dalitz plot approximation) we have relation (9) (where $\cos^2\varphi$ is to be interpreted as a weighted average: $0 \leqslant \cos^2\varphi \leqslant 1$) and

$$I_-'^{(000)} = \tfrac{3}{2} I_-'^{(+-0)}, \tag{15}$$

$$|I_-'^{(+-0)}| \leqslant 2[\Gamma_2'^{(+-0)}(2\Gamma_+'^{(+00)} - \Gamma_2'^{(+-0)})]^{\frac{1}{2}}. \tag{16}$$

Equation (16) becomes an equality in the approximation of a single constant $I = 1$ amplitude.

In comparing inequalities (14) and (16) with experiment [15] one must take into account the fact that the electromagnetic corrections to the right-hand sides are expected to be as important as the effects of CP violation. With the present experimental uncertainties on the widths [16], it would require a

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[14] See for example, J. S. BELL and J. STEINBERGER: *Proc. Oxford Int. Conf. on Elem. Part.* (1965), p. 195.

[15] K_L and K_S are not orthogonal states; as a consequence, the experimental value for Γ_L ($= \Gamma_2$ to first order in the CP violation) depends on whether it is determined from a beam which is initially K^0 or $\overline{K^0}$: to first order in ε, for $t \gg 1/\Gamma_S$:

$$|a_{K^0}(t)|^2 = |a_2|^2 \exp[-\Gamma_L t](1 - \operatorname{Re}\varepsilon)/2,$$
$$|a_{\overline{K^0}}(t)|^2 = |a_2|^2 \exp[-\Gamma_L t](1 + \operatorname{Re}\varepsilon)/2.$$

However the error introduced by neglecting this normalization factor is less than the experimental accuracy one can hope to obtain.

[16] See, for example, J. P. LOWYS: *Thèse de Doctorat d'Etat*, Orsay, No. 185 (1966) for a recent compilation of data. See also: L. BEHR, V. BRISSON, P. PETIAU, E. BELLOTTI, A. PULLIA, M. BALDO CEOLIN, E. CALIMANI, S. CIAMPOLILLO, H. HUZITA, A. SCONZA, B. AUBERT, L. M. CHOUNET, J. P. LOWYS and C. PASCAUD: *Phys. Lett.*, **22**, 540 (1966) for a recent study of the $K^0 \to 3\pi$ rate as a function of time. Here it is found that the results are compatible with the assumption $|\Delta I| = \frac{1}{2}$ for both CP-violating and CP-conserving amplitudes. Assuming $|\Delta I| = \frac{1}{2}$ and a constant CP-conserving phase, these authors find $\varphi = -(19^{+9}_{-33})$ degrees. However a larger violation with $|\Delta I| > \frac{1}{2}$ cannot be ruled out at present.

rather large violation of CP, $|I_-^{(k)}|/2\Gamma_L^{(k)} \gtrsim 0.5$, to rule out the $|\Delta I| = \frac{1}{2}$ rule on this basis. To improve this limit, an accurate determination of Γ_L from a pure K_L beam is desirable, although one cannot hope to set a better limit than about five per cent on this quantity. On the other hand a value of $|I_+^{(k)}|/2\Gamma_L^{(k)} \gtrsim$ a few per cent would probably indicate the presence of CP-violating amplitudes with $|\Delta I| > \frac{1}{2}$.

In contrast, relations between quantities directly dependent on a CP violation are not subject to this criticism. For example, eq. (15) is expected to be as good as the analogous relation

$$\Gamma_L'^{(000)} = \tfrac{3}{2}\Gamma_L'^{(+-0)}, \tag{17}$$

which is verified experimentally with a twenty per cent error [16].

Note that, like (17), eq. (15) tests only the rule $|\Delta I| < \frac{5}{2}$.

2·3. *Amplitudes with* $|\Delta I| > \frac{3}{2}$. – The final state $I = 3$ may arise from $|\Delta I| > \frac{3}{2}$ transitions. In this case I_+ may be significant, provided that there are large differences in the $I = 3$ and $I = 1$ final-state interaction phase shifts. The $I = 3$ amplitude is totally symmetric and only the totally symmetric part of the $I = 1$ amplitude contributes to the interference term. Approximating these amplitudes by constants, one has

$$I_+^{(k)} = -2\, A_3^{(k)} A_1^{(k)} \sin(\delta_3 - \delta_1) \sin(\varphi_3 - \varphi_1),$$

$$I_-^{(k)} = \quad 2iA_3^{(k)} A_1^{(k)} \cos(\delta_3 - \delta_1) \sin(\varphi_3 + \varphi_1),$$

where φ_I and δ_I are the CP-violating phases and the final-state phase shifts respectively. As $A_3 \ll A_1$, this amplitude will be important only if $\varphi_3 \gg \varphi_1$, so that

$$I_+^{(k)}/I_-^{(k)} \approx i \operatorname{tg}(\delta_3 - \delta_1). \tag{18}$$

We have the further relation for $I = 3$ final states

$$I_-^{(000)} = -I_-^{(+-0)} \tag{19}$$

to be compared with (15) for $I = 1$ final states.

3. – $K^{\pm} \to 3\pi$.

If CP is not conserved, the partial rates for K and $\overline{K}$ in 3π decay may differ: $\Gamma_+'^{(k)} \neq \Gamma_-'^{(\bar{k})}$, although CPT requires

$$\sum_k \Gamma_+^{(k)} = \sum_k \Gamma_-^{(\bar{k})}. \tag{20}$$

As pointed out by UEDA and OKUBO [17], the differences between partial decay rates vanish in the approximation of a linear Dalitz plot if there is no $I = 3$ in the final state. We define

$$\Delta^{(+00)} = \Gamma_+^{\prime(00+)} - \Gamma_-^{\prime(00-)}, \tag{21}$$

$$\Delta^{(++-)} = \Gamma_+^{\prime(++-)} - \Gamma_-^{\prime(--+)}, \tag{22}$$

$$\Delta^{(+00)} + \Delta^{(++-)} = 0. \tag{23}$$

In Sect. **4**, we show that if the $\Delta^{(k)}$ are nonvanishing, they are related to the $I^{\pm}$ according to isotopic-spin properties. We separate contributions to the $I^{\pm}$ and $I^{\pm}$ according to the isotopic spin I of the final state, and the isospin of the CP-violating transition $\Delta T = I \pm \frac{1}{2}$:

$$\Delta^{(k)} = \sum_{I,T} \Delta^{(k)}(I, \Delta T), \tag{24}$$

$$I_\pm^{\prime(k)} = \sum_{I,T} I_\pm^{\prime(k)}(I, \Delta T), \tag{25}$$

$$\sum_k \Delta^{(k)}(I, \Delta T) = \sum_k I_+^{\prime(k)}(I, \Delta T) = 0. \tag{26}$$

For each value of I and T one has the relation

$$I_+^{\prime(+-0)}(I, \Delta T) = C(I, \Delta T)\Delta^{(+00)}(I, \Delta T). \tag{27}$$

The coefficients C are given in Table I, where we also summarize the results of Sect. **2**.

TABLE I.

$$I_+^{\prime+-0}(I, \Delta T) + I_+^{\prime 000}(I, \Delta T) = \Delta^{+00}(I, \Delta T) + \Delta^{++-}(I, \Delta T) = 0.$$

ΔT	I	$\lvert I_-^{\prime+-0}\rvert$	$\dfrac{I_-^{\prime 000}}{I_-^{\prime+-0}}$	$\dfrac{I_+^{\prime+-0}}{I_-^{\prime+-0}}$	$\dfrac{I_+^{+-0}}{\Delta^{+00}}$	
$\frac{1}{2}$	1	$\leqslant 2[(2\Gamma^{\prime 00} - \Gamma^{\prime+-0})\Gamma^{\prime+-0}]^{\frac{1}{2}}$	$\approx \frac{3}{2}$	$\leqslant 0.05$	1	
$\frac{3}{2}$	1	?	$\approx \frac{3}{2}$	$\leqslant 0.05$	-2	$\lvert\Delta(2, \frac{3}{2})/\Delta(1, \frac{3}{2})\rvert \sim 1$
	2	0	—	—	0	
$\frac{5}{2}$	2	0	—	—	0	$\lvert\Delta(2, \frac{5}{2})/\Delta(3, \frac{5}{2})\rvert \leqslant 0.05$
	3	?	-1	$\approx \mathrm{tg}\,(\delta_3 - \delta_1)$	$-\frac{3}{4}$	
$\frac{7}{2}$	3	?	-1	$\approx \mathrm{tg}\,(\delta_3 - \delta_1)$	1	

[17] Y. UEDA and S. OKUBO: *Phys. Rev.*, **139**, B 1591 (1965).

Note that for $I = 1$ the relevant terms are interference terms between the CP-violating amplitude and the dominant $\Delta I = \frac{1}{2}$ CP-conserving amplitude. For K^0 decays a term with $I = 2$ may contribute, but $\Delta T \sim 3\pi/2$, $\sin\Delta T \sim 1$, so this term arises from the integral of a quadratic function in the pion energies and is not expected to be more important than the corresponding contributions to the $I = 1$ amplitude. Thus for $\Delta T = \frac{\pi}{2}$, $I(2, \frac{\pi}{2})$ may be as important as $I(1, \frac{\pi}{2})$, but for $\Delta T = \frac{3\pi}{2}$, $I(3, \frac{\pi}{2})$ is expected to dominate.

A further analysis in terms of isospin amplitudes may be achieved by studying differences in K^+ and K^- decay distributions, as is discussed in ref. (7).

4. – Derivations.

To derive the results presented in Sect. 2 and 3 we use a [hypothetical] set of states n which diagonalize the S matrix. We separate states according to charge and isotopic spin, with the following phase conventions:

$$I_3 |n, I, -\rangle = -|n, I, -\rangle ,$$

$$I_3 |n, I, 0\rangle = 0 ,$$

$$I_+ |n, I, 0\rangle = \sqrt{I(I+1)}\, |n, I, +\rangle ,$$

$$\exp[i\pi I_2] |n, I, -\rangle = |n, I, +\rangle = CP |n, I, -\rangle ,$$

$$\exp[i\pi I_2] |n, I, 0\rangle = (-)^I |n, I, 0\rangle = CP |n, I, 0\rangle ,$$

$$K^0 = CP \bar{K}^0 ,$$

$$K^- = CP \bar{K}^- .$$

Then for the T matrix amplitudes, CPT requires

$$\langle n^0, I\ T\ \bar{K}^0\rangle = \exp[2i\delta_I^n] \langle n^0, I\ T\ K^0\rangle^* = \exp[i\delta_I^n] A_I^n , \tag{28}$$

$$\langle n^0, I\ T\ K^0\rangle = (-)^I \exp[2i\delta_I^n] \langle n^0, I\ T\ \bar{K}^0\rangle^* . \tag{29}$$

We can also define the amplitudes

$$\langle n^0, I\ T\ K^0_{1,2}\rangle = \exp[i\delta_I^n] A^{1,n}_{(1,2)} , \tag{30}$$

$$K_1 = [K - \bar{K}]/\sqrt{2} , \qquad K_2 = [K + \bar{K}]/\sqrt{2} ,$$

with the phase convention defined by eq. (8), $K_2 \to 2\pi (I = 0)$.

The decay amplitudes as functions of the pion energies $\{\omega\}$ will be given by

$$\langle \omega(k)|T|K_i\rangle = \sum_I a_i^{I(k)}(\omega)\,, \tag{31}$$

$$a_i^{I(k)}(\omega) = \sum_n f_n^{I(k)}(\omega)\exp[i\delta_{I,n}]A_i^{I,n} \equiv \sum_n F_n^{I(k)}(\omega)A_i^{I,n}\,, \tag{32}$$

$$f_n^{I(k)}(\omega) = \langle \omega(k)|I, n, I_3\rangle \delta_{I_3,I_3(k)}\,. \tag{33}$$

The functions $f_n^{I(k)}(\omega)$ (and consequently the functions $F_n^{I(k)}(\omega)$) have the orthogonality property

$$\sum_{(k)} \int d\omega\, f_n^{I(k)}(\omega) f_{n'}^{I'(k)*}(\omega) = \delta_{nn'}\delta_{II'}\,. \tag{34}$$

We consider first transitions with $|\Delta I| = \frac{1}{2}$; the K^+ and K^0 amplitudes are related by Clebsch-Gordan coefficients:

$$\begin{cases} A_+^{1,n} \equiv a_{\frac{1}{2}}^n \exp[i\alpha_{\frac{1}{2}}^n] = (A_-^{1,n})^*\,, \\ A_2^{1,n} = a_{\frac{1}{2}}^n \cos\alpha_{\frac{1}{2}}^n\,, \\ A_0^{1,n} = i a_{\frac{1}{2}}^n \sin\alpha_{\frac{1}{2}}^n\,. \end{cases} \tag{35}$$

The $a_{\Delta I}^n$ are real numbers, and the $\alpha_{\Delta I}^n$ are CP-violating phases. For the decay rates we have

$$\Gamma_i^{(k)} = \int d\omega |a_i^{1(k)}(\omega)|^2 - \sum_{n,m} \int d\omega\, F_n^{1(k)}(\omega) F_m^{1(k)*}(\omega) A_i^{1,n}(A_i^{1,m})^*\,. \tag{36}$$

Summing over charge modes, and using condition (34) one obtains

$$\sum_k \Gamma_i^{(k)} = \sum_n |A_i^{1,n}|^2\,. \tag{37}$$

Finally, using the amplitudes (35),

$$\begin{cases} \Gamma_-^{(+00)} + \Gamma_-^{(++-)} = \sum_n (a_{\frac{1}{2}}^n)^2\,, \\ \Gamma_L^{(000)} + \Gamma_L^{(+-0)} = \sum_n (a_{\frac{1}{2}}^n)^2 \cos^2\alpha_{\frac{1}{2}}^n\,, \end{cases} \tag{38}$$

giving the inequality (16).

Now, the F_n^i are related for different charge modes [18]:

$$F_n^{1(+00)}(\omega_0, \omega_0', \omega_+) = \frac{1}{\sqrt{2}} F_n^{1(+-0)}(\omega_+, \omega_-, \omega_0) \tag{39}$$

and

$$\int \mathrm{d}\omega \left| \sum_n a_n F_n^{(000)} \right|^2 = \int \mathrm{d}\omega \left| \sum_n a_n F_n^{(++-)} \right|^2 - \int \mathrm{d}\omega \left| \sum_n a_n F_n^{(+00)} \right|^2 . \tag{40}$$

So that if the phase $\alpha_{\frac{1}{2}}$ is *constant* (independent of n), we get relation (9) ($\alpha_{\frac{1}{2}} = \varphi$).

The $F_n^1(\omega)$ may be separated into symmetric and nonsymmetric parts which do not interfere in the integrated rates. If we may neglect the nonsymmetric contribution (which is at least quadratic in pion energies), we have the further relations

$$\begin{cases} F_n^{1(000)}(\omega) \approx \sqrt{\frac{3}{2}} F_n^{1(+-0)}(\omega) , \\ F_n^{1(++-)}(\omega) \approx 2 F^{1(+00)}(\omega) . \end{cases} \tag{41}$$

Then, using condition (34),

$$\begin{cases} \int \mathrm{d}\omega F_n^{1(000)}(\omega) F_m^{1(000)*}(\omega) \approx \frac{3}{2} \int \mathrm{d}\omega F_n^{1(+-0)}(\omega) F_m^{1(+-0)*}(\omega) \approx \frac{3}{5} \delta_{mn} , \\ \int \mathrm{d}\omega F_n^{1(++-)}(\omega) F_m^{1(++-)*}(\omega) \approx 4 \int \mathrm{d}\omega F_n^{1(+00)}(\omega) F_m^{1(+00)*}(\omega) \approx \frac{4}{5} \delta_{mn} . \end{cases} \tag{42}$$

Inserting this result in eq. (30), we get again relation (9) with $\cos^2\varphi = \sum_n (a_{\frac{1}{2}}^n \cos\alpha_{\frac{1}{2}}^n)^2 / \sum_n (a_{\frac{1}{2}}^n)^2 \leqslant 1$.

Now consider the interference terms $I_\pm$ for $I = 1$ final states: for $|\Delta I| = \frac{1}{2}$ transitions

$$I_\pm^{(k)}(1, \tfrac{1}{2}) = \sum_{m,n} i \int \mathrm{d}\omega F_n^{(k)} F_m^{(k)*} a_n^{\frac{1}{2}} a_m^{\frac{1}{2}} (\sin\alpha_n^{\frac{1}{2}} \cos\alpha_m^{\frac{1}{2}} \mp \cos\alpha_n^{\frac{1}{2}} \sin\alpha_m^{\frac{1}{2}}) = \tag{43}$$

$$= \sum_{m,n} \int \mathrm{d}\omega F_n^{(k)} F_m^{(k)*} \sin(\alpha_n^{\frac{1}{2}} \mp \alpha_m^{\frac{1}{2}}) a_n^{\frac{1}{2}} a_m^{\frac{1}{2}} .$$

Clearly, for $\alpha_{\frac{1}{2}}$ constant, $I_+^{(k)}$ vanishes, and

$$I_-^{(k)} = 2i \cos\alpha_{\frac{1}{2}} \sin\alpha_{\frac{1}{2}} \sum_{m,n} \int \mathrm{d}\omega F_n^{(k)} F_m^{(k)*} a_n^{\frac{1}{2}} a_m^{\frac{1}{2}} = 2i \operatorname{tg}\alpha_{\frac{1}{2}} I_2^{(k)} . \tag{44}$$

[18] R. F. Sawyer and K. C. Wali: *Nuovo Cimento*, **17**, 938 (1960).

Making the substitution (eq. (42))

$$\int d\omega F_n^{(k)} F_m^{(k)*} \to \chi^{(k)} \delta_{nm} ,$$

we have

$$\begin{aligned} I_+^{(k)} &= 0 , \\ I_-^{(k)} &= 2i\chi^{(k)} \sum_n (a_n^1)^2 \cos\alpha_n^1 \sin\alpha_n^1 . \end{aligned} \tag{45}$$

One can show that

$$\Big(\sum_n (a_n)^2 \cos\alpha_n \sin\alpha_n\Big)^2 \leqslant \Big(\sum_n (a_n)^2 \cos^2\alpha_n\Big)\Big(\sum_n (a_m)^2 \sin^2\alpha_n\Big) . \tag{46}$$

Then, neglecting nonsymmetric terms,

$$|\tfrac{2}{3} I_-^{(000)}|^2 = |I_-^{(+-0)}|^2 \leqslant 4 I_2^{(+-0)} [2 I_+^{(+00)} - I_2^{(+-0)}] , \tag{47}$$

giving relations (15) and (16). If one includes contributions from nonsymmetric terms, one has the less restrictive relation (14). Note that, by virtue of eq. (34),

$$\sum_{(k)} I_+^{(k)} = 0 . \tag{48}$$

Now consider the general form of the interference term; in the notation of Sect. 3

$$I_\pm^{(k)} = \sum_{I, \Delta T} I_\pm^{(k)}(I, \Delta T) , \tag{49}$$

we define the various isospin amplitudes:

$$\left| \begin{aligned} A_2^{1,n} &= a_n^{\frac{1}{2}} \cos\alpha_n^{\frac{1}{2}} + a_n^{\frac{3}{2}} \cos\alpha_n^{\frac{3}{2}} , \\ A_1^{1,n} &= i(a_n^{\frac{1}{2}} \sin\alpha_n^{\frac{1}{2}} + a_n^{\frac{3}{2}} \sin\alpha_n^{\frac{3}{2}}) , \\ A_+^{1,n} &= a_n^{\frac{1}{2}} \exp[i\alpha_n^{\frac{1}{2}}] - \tfrac{1}{2} a_n^{\frac{3}{2}} \exp[i\alpha_n^{\frac{3}{2}}] = (A_-^{1,n})^* , \\ A_2^{3,n} &= b_n^{\frac{5}{2}} \cos\beta_n^{\frac{5}{2}} + b_n^{\frac{7}{2}} \cos\beta_n^{\frac{7}{2}} , \\ A_1^{3,n} &= i(b_n^{\frac{5}{2}} \sin\beta_n^{\frac{5}{2}} + b_n^{\frac{7}{2}} \sin\beta_n^{\frac{7}{2}}) , \\ A_+^{3,n} &= \sqrt{\tfrac{2}{3}} (b_n^{\frac{5}{2}} \exp[i\beta_n^{\frac{5}{2}}] - \tfrac{3}{4} b_n^{\frac{7}{2}} \exp[i\beta_n^{\frac{7}{2}}]) . \end{aligned} \right. \tag{50}$$

In addition to expression (43) for $I^{(k)}_{+}(1, \frac{1}{2})$, one obtains, to first order in $\Delta I = \frac{1}{2}$ amplitudes,

$$I^{(k)}_{+}(1, \tfrac{3}{2}) = \sum_{m,n} i \int d\omega (F^{1(k)}_{n} F^{1(k)*}_{m} - F^{1(k)}_{m} F^{1(k)*}_{n}) a^{\frac{3}{2}}_{n} a^{\frac{1}{2}}_{m} \sin(\alpha^{\frac{3}{2}}_{n} - \alpha^{\frac{1}{2}}_{m}) , \tag{51}$$

$$I^{(k)}_{+}(3, \Delta T) = \sum_{m,n} i \int d\omega (F^{3(k)}_{n} F^{1(k)*}_{m} - F^{1(k)}_{m} F^{3(k)*}_{n}) b^{\Delta T}_{n} a^{\frac{1}{2}}_{m} \sin(\beta^{\Delta T}_{n} - \alpha^{\frac{1}{2}}_{m}) . \tag{52}$$

Equation (34) gives immediately

$$\sum_{(k)} I^{(k)}_{\pm}(3, \Delta T) = \sum_{(k)} I^{(k)}_{+}(1, \Delta) = 0 , \tag{53}$$

whereas the $I^{(k)}_{-}(1, \Delta T)$ should approximately obey relation (15).

The differences in partial rates for $K^{\pm}$ decay are given by

$$\Delta^{(k)} = \int d\omega (|a^{(k)}_{+}|^2 - |a^{(\bar{k})}_{-}|^2) . \tag{54}$$

Writing

$$\Delta^{(k)} = \sum_{I, \Delta T} \Delta^{(k)}(I, \Delta T) ,$$

$$F^{I(k)}(\omega_a, \omega_b, \omega_c) = F^{I(\bar{k})}(\omega_{\bar{a}}, \omega_{\bar{b}}, \omega_{\bar{c}}) ,$$

one obtains, using the amplitudes given in (50),

$$\Delta^{(k)}(1, \tfrac{1}{2}) = \sum_{m,n} \int d\omega F^{1(k)}_{n} F^{1(k)*}_{m} a^{\frac{1}{2}}_{n} a^{\frac{1}{2}}_{m} [\exp[i(\alpha^{\frac{1}{2}}_{n} - \alpha^{\frac{1}{2}}_{m})] - \exp[i(\alpha^{\frac{1}{2}}_{m} - \alpha^{\frac{1}{2}}_{n})]]$$
$$= 2i \sum_{m,n} \int d\omega F^{1(k)}_{n} F^{1(k)*}_{m} a^{\frac{1}{2}}_{n} a^{\frac{1}{2}}_{m} \sin(\alpha^{\frac{1}{2}}_{n} - \alpha^{\frac{1}{2}}_{m}) , \tag{55}$$

$$\Delta^{(k)}(1, \tfrac{3}{2}) = -i \sum_{m,n} \int d\omega [F^{1(k)}_{n} F^{1(k)*}_{m} - F^{1(k)}_{m} F^{1(k)*}_{n}] a^{\frac{3}{2}}_{n} a^{\frac{1}{2}}_{m} \sin(\alpha^{\frac{3}{2}}_{n} - \alpha^{\frac{1}{2}}_{m}) , \tag{56}$$

$$\Delta^{(k)}(3, \tfrac{5}{2}) = 2i\sqrt{\tfrac{2}{3}} \sum_{m,n} \int d\omega [F^{3(k)}_{n} F^{1(k)*}_{m} - F^{1(k)}_{m} F^{3(k)*}_{n}] b^{\frac{5}{2}}_{n} a^{\frac{1}{2}}_{m} \sin(\beta^{\frac{5}{2}}_{n} - \alpha^{\frac{1}{2}}_{m}) , \tag{57}$$

$$\Delta^{(k)}(3, \tfrac{7}{2}) = -i\sqrt{\tfrac{3}{2}} \sum_{m,n} \int d\omega [F^{3(k)}_{n} F^{1(k)*}_{m} - F^{1(k)}_{m} F^{3(k)*}_{n}] b^{\frac{7}{2}}_{n} a^{\frac{1}{2}}_{m} \sin(\beta^{\frac{7}{2}}_{n} - \alpha^{\frac{1}{2}}_{m}) . \tag{58}$$

These quantities may be related to the I_{+} by using eq. (39) and the analogous relation

$$F^{(3)(+00)}_{n}(\omega_+, \omega_0, \omega'_0) = -2/\sqrt{3} \bar{F}^{(3)(+-0)}(\omega_+, \omega_-, \omega_0) . \tag{59}$$

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On comparison with eqs. (43), (51) and (52)

$$
(60)\qquad \left\{
\begin{aligned}
I_{+}^{(+-0)}(1, \tfrac{1}{2}) &= \quad A^{(+00)}(1, \tfrac{1}{2}), \\
I_{+}^{(+-0)}(1, \tfrac{3}{2}) &= -2A^{(+00)}(1, \tfrac{3}{2}), \\
I_{+}^{(+-0)}(3, \tfrac{5}{2}) &= -\tfrac{3}{4}A^{(+00)}(3, \tfrac{5}{2}), \\
I_{+}^{(+-0)}(3, \tfrac{7}{2}) &= A^{(+00)}(3, \tfrac{7}{2}).
\end{aligned}
\right.
$$

In the above considerations we have neglected electromagnetic mass differences. These should of course be taken into account in the phase space integrals. It is difficult to estimate their contribution to the invariant amplitudes: one could hope that they would give only a small correction to the terms quadratic in pion energies (which are present in the symmetric as well as nonsymmetric amplitudes), and thus a correction to the total amplitude no greater than the over-all electromagnetic effects.

5. – Conclusions.

If the real part of the $K^0 \to 3\pi$ interference term is significant, one can probably infer that the CP-violating amplitudes with $|\Delta I| > \frac{1}{2}$ are important. In particular if $I \sim I'$, this implies *either* large cancellations between $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ amplitudes or the presence of $\Delta I > \frac{3}{2}$ amplitudes (or both). This conclusion can be further tested by comparison with the difference between K_+ and K_- partial rates.

If only the imaginary part of the interference term is important (the most probable case for $\Delta I < \frac{5}{2}$) one can in principle distinguish between $I = 1$ and $I = 3$ final states by comparing $K^0 \to \pi^+\pi^-\pi^0$ with $K^0 \to \pi^0\pi^0\pi^0$. However, if $|\Delta I| < \frac{3}{2}$ it will not be possible to test the $|\Delta I| = \frac{1}{2}$ rule in the CP-violating amplitude from total rates alone, unless the $\Delta I = \frac{3}{2}$ amplitude is greater than the upper limit one can deduce for $|\Delta I| = \frac{1}{2}$.

* * *

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RIASSUNTO (*)

Si analizzano, in funzione dei parametri dello spin isotopico, gli effetti che violano la *CP* nel decadimento $K \to 3\pi$. In particolare si studiano il termine di interferenza fra le ampiezze del K_S e del K_L e la differenza delle quantità parziali nel decadimento del $K^\pm$. In generale ci si attende che la parte reale del termine di interferenza sia molto minore della parte immaginaria per transizioni con $|\Delta I| < \frac{5}{2}$. Il successo della regola $|\Delta I| = \frac{1}{2}$ nel decadimento del K_L e del K^+ pone un limite superiore alla parte immaginaria del termine di interferenza per ampiezze che violano la *CP* con $|\Delta I| = \frac{1}{2}$. Questo limite probabilmente è troppo alto per essere utile, ma un effetto significativo nella parte reale indicherebbe probabilmente che ampiezze con $|\Delta I| > \frac{1}{2}$ sono importanti. Si può verificare questa conclusione confrontando la parte reale del termine di interferenza con le differenze delle quantità parziali del decadimento $K^\pm$; i due effetti sono in relazione diretta tramite coefficienti che dipendono da $|\Delta I|$ e lo spin isotopico dello stato finale. Nel decadimento del K^0, si possono distinguere gli stati finali $I = 1$ ($|\Delta I| < \frac{5}{2}$) dagli stati finali $I = 3$ ($|\Delta I| \geqslant \frac{5}{2}$) confrontando le parti immaginarie dei termini di interferenza nei modi $\pi^+\pi^-\pi^0$ e $3\pi^0$.

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Изотопический спиновый анализ возможных эффектов *CP* нарушения в $K \to 3\pi$.

Резюме (*). — В терминах парамегров изогопического спина анализируются эффекты *CP* нарушения в распаде $K \to 3\pi$. В частности, рассматриваются интерференционный член между K_S и K_L амплитудами и разность в парциальных скоростях для $K^\pm$ распада. Вообще, ожидается, что реальная часть интерференционного члена значительно меньше мнимой части для переходов с $|\Delta I| < \frac{5}{2}$. Успех правила $|\Delta I| = \frac{1}{2}$ в K_L и $K^\pm$ распаде устанавливает верхний предел для мнимой части интерференционного члена для *CP* нарушаюших амплитуд с $|\Delta I| = \frac{1}{2}$. Этот предел, вероятно, слишком высох, чтобы быть полезным, но существенный эффект в реальной части, вероятно будет указывать, что амплитуды с $|\Delta I| > \frac{1}{2}$ являются важными. Это заключение может быть проверено посредством сравнения реальной части интерференционного члена с разностями в парциальных скоростях для $K^\pm$ распада; указанные два эффекта непосредственно связаны через коэффициенты, зависящие от $|\Delta I|$ и изотопического спина конечного состояния. В K^0 распаде конечные состояния $I = 1$ ($|\Delta I| < \frac{5}{2}$) можно отличить от конечных состояний $I = 3$ ($|\Delta I| > \frac{5}{2}$) путем сравнения мнимых частей интерференционных членов в схемах $\pi^+\pi^-\pi^0$ и $3\pi^0$.

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